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On the cosmological implications and related issues in string theories

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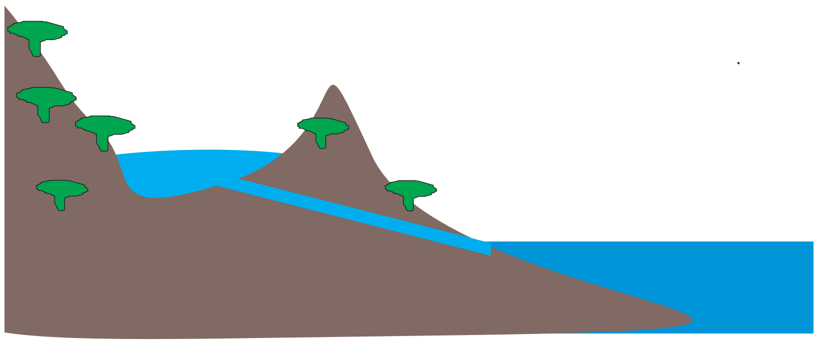


Outline of the present Talk

- ▲ Cosmology and Einstein Gravity: a few facts
- ▲ Strings and Branes
- ▲ *Moduli Fields in Effective String Theory Models*
- ▲ Quantum corrections and de Sitter Space
- ▲ Inflation in String Theory Models
- ▲ Concluding Remarks

A few facts about Cosmology

▲ *Major Observational Discovery* $\sim$ 22 *years ago:*

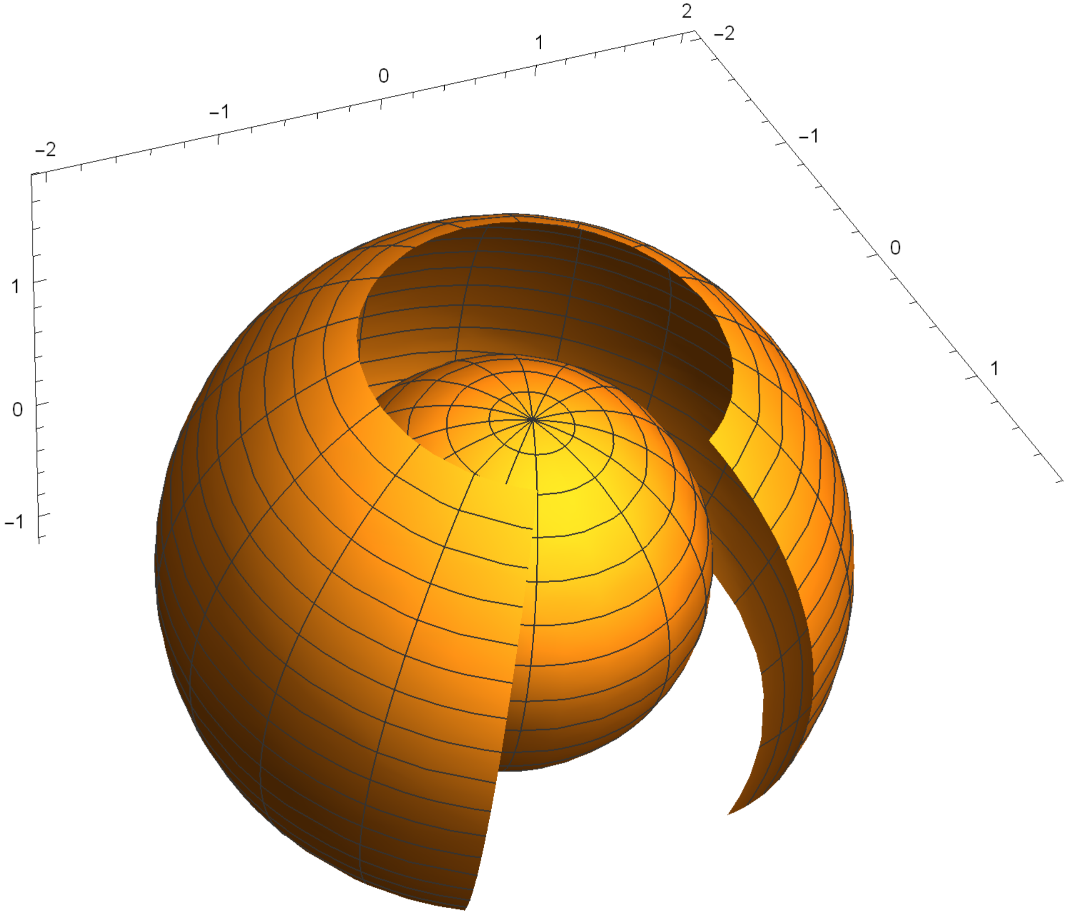
Accelarating Expansion of the Universe

Against our intuition

▲ **What do we mean by 'expansion'?**

• Galaxies move away from each other, $v \propto$ distance

• Intrinsic expansion: it's the scale of space that changes
(*think of two ants on the surface of an inflating balloon*)



2-d Expansion: only surface augments, objects on it do not move

Content of the Universe

▲ 5% Visible Matter

▲ 27% **Dark Matter**

explains gravitational & astrophysical phenomena

(*holds stars together in galaxies ... etc*)

No or feeble interaction with visible matter $\Rightarrow$ **undetectable?**

▲ 68% **Dark Energy**

$\Rightarrow$ Explains Accelerated Expansion *of the Universe*

▲ **Dark Energy (DE)** : **In Einstein's Gravity** ▲

DE *is expressed with a positive cosmological constant:*

$$\Lambda \approx 10^{-120} M^4_{Planck} \; (\sim m^4_\nu)$$

▲ General Relativity (GR) Equations:▲

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda \, g_{\mu\nu} = \kappa \, T_{\mu\nu}$$

- $R_{\mu\nu} \to$ Ricci tensor
- $R \to$ Curvature scalar
- $T_{\mu\nu} \to$ Energy-Momentum tensor

Reminder

throughout the present talk we will often refer to the

Principle of Least Action:

For a physical system described by the Lagrangian

$$\mathcal{L}(q, \dot{q}) = T - U$$

the system follows the 'path' for which the action

$$\mathcal{S} = \int_{t_i}^{t_f} \mathcal{L}(q, \dot{q}) dt$$

is stationary

$$\delta\mathcal{S} = 0 \ \Rightarrow$$

$\Rightarrow$ Equations of Motion, Conservation Laws...

Dark Energy (DE): Significant implications in GR

▲ Consider the action in field theory case (ϕ scalar)

$$S = \int d^4x\, \mathcal{L}(\phi)$$

$$\mathcal{L}(\phi) = \partial_\mu \phi \partial^\mu \phi - U(\phi)$$

Only differences of (potential) energy matter!

▲ Action in Einstein Theory

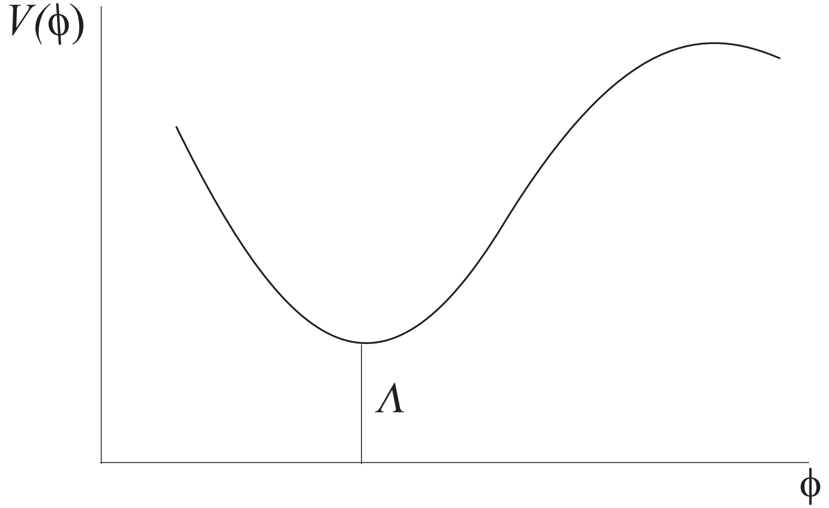
$$S = \frac{1}{\kappa} \int d^4x \sqrt{-g} \left(\frac{1}{2} R - \Lambda + \kappa \mathcal{L}(\phi) \right)$$

Adding a constant Λ implies a term $\sqrt{-g}\,\Lambda$ in the Lagrangian.
Change of $U(\phi)$ modifies the vacuum energy!

▲ **DE:** Equivalent Effective Field Theory description▲

Potential Energy $V(\phi)$ of a scalar field, ϕ

(*... for example ϕ could be the Higgs field...*)



We live in a Universe with positive cosmological constant

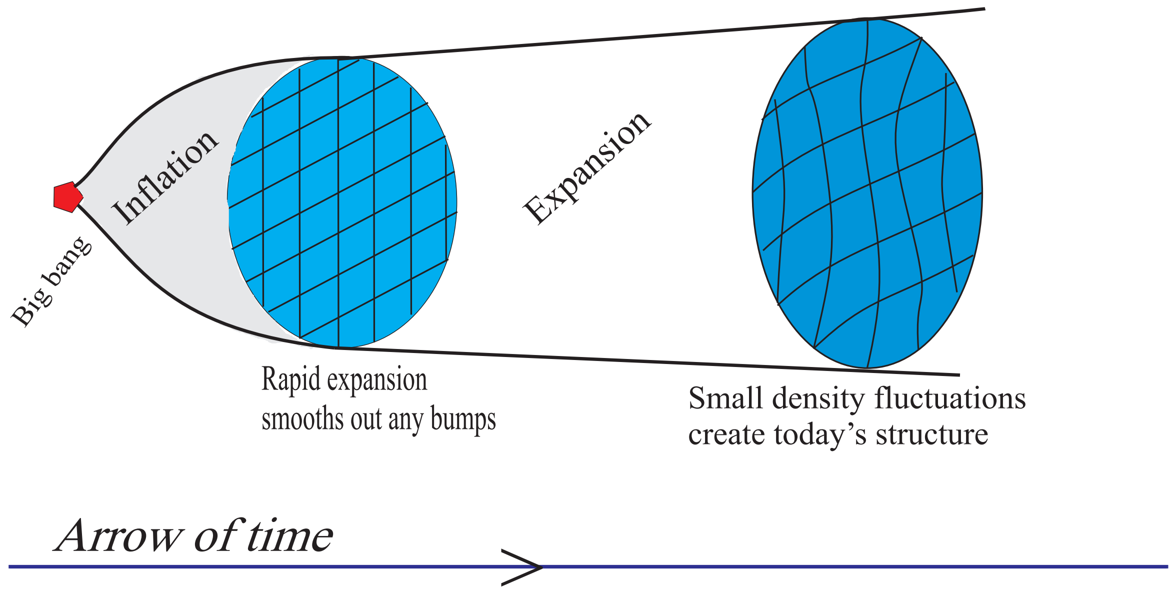
$$\Lambda > 0$$

▲ de Sitter vacua ▲

With a few additional requirements:

$V(\phi)$ appropriate for **Cosmological Inflation** (CI)

(*CI solves problems of flatness, horizon, monopoles,...*)



▲ **Challenge :**

Embedment of Inflationary Scenario in String Theory

STRING THEORY

$\Downarrow$

String Theory unifies Einstein's theory of gravity with quantum theory

This is a highly non trivial accomplishment!

▲ String Theory predicts SIX **new dimensions**!▲

these must be tiny to escape detection! this is called

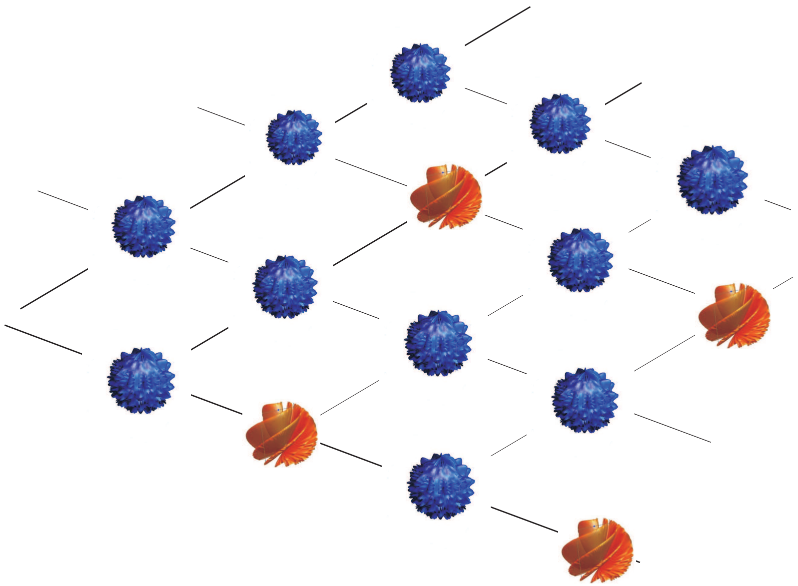
Compactification

Ten dimensional space must look like:

$$\mathcal{R}^{3,1} \times \mathcal{X}_6$$

$\mathcal{R}^{3,1}$ Minkowski spacetime

$\mathcal{X}_6$ Six compactified dimensions



▲ Pictorial compactification of the six extra dimensions and the "lattice" of 4-d spacetime

▲ In principle, geometry of compactification can differ from point to point (*compactification on sphere, torus...*)

▲ Physical properties of 4-d spacetime depend on the choice of compactification

String Theory Spectrum

Open Strings

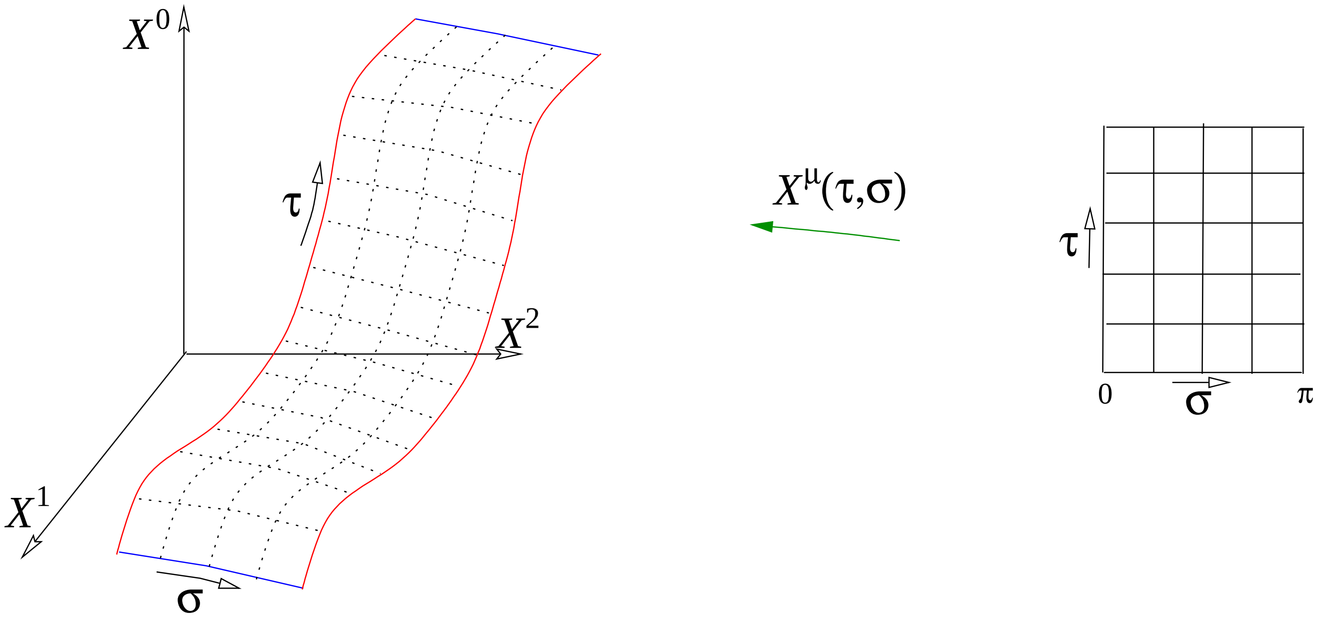
Closed Strings

Branes (*generalized notion of membranes*)

Moduli fields

... a string in D-dimensions sweeps a worldsheet

$$X^{\mu}(\tau, \sigma),\ \mu = 0, 1, ..., D-1$$



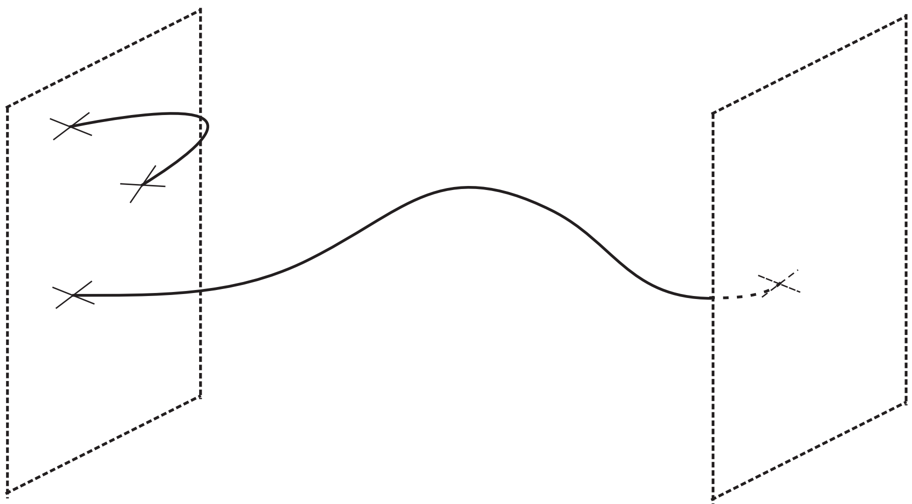
Strings have lengths

$$0 \leq \sigma \leq 2\pi$$

Configurations of strings and D-branes

$$\Downarrow$$

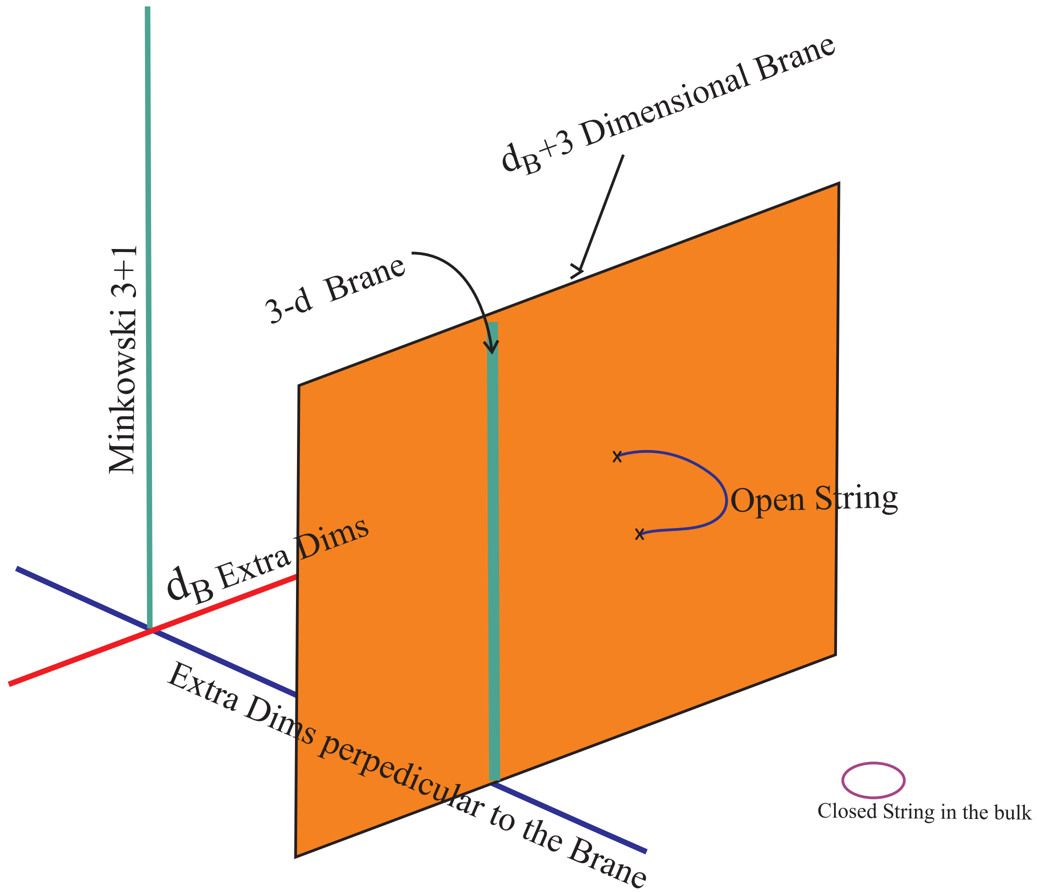
A *Dp*-brane is an object with *p*-spatial dimensions



Open string endpoints on D-branes

STRING-BRANE Unification Scenario...

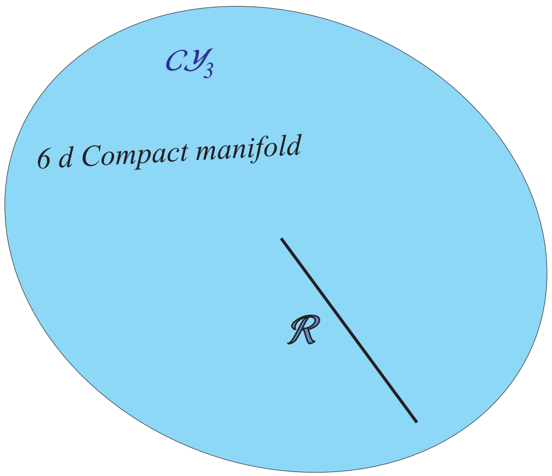
QFT (particles ∼ open strings), Gravity (closed strings propagating in the bulk → explains weakness of gravity)



Compactifying the six additional dimensions, we are left with an **Effective Field Theory (EFT)** *to describe the four-dimensional world*

▲ **Compactifications** characterised by **large** # of moduli: ▲

▲ Example: **Compactification on Calabi-Yau (CY)** manifold



$\mathcal{CY}_3$ characterised by radius R (**modulus**). However.... $\rightarrow$

Classical EoM invariant under R-rescalings

$$\Downarrow$$

$\forall$ solution leading to Effective Field Theory (SM, GUT,...)

$\exists$ **family of solutions** by changing R.

$\Rightarrow R$ must be fixed! $\Leftarrow$

▲ at **Effective Field Theory** level: ▲

... *in general*

▲ Deformations of **Compactifications** correspond to **massless scalar fields** in four dimensions and if they couple to Standard Model fields they create problems with fifth forces and other cosmological issues...

▲ **Tasks** ▲

▲ In our EFT model we must guarantee positive mass-squared for **all moduli fields**

$\Rightarrow$

$\Rightarrow$ $\mathcal{M}$*oduli* $\mathcal{S}$*tabilisation* $\Leftarrow$

Also, in order to solve the cosmological problems

▲ Look for possible **Inflaton** candidates among *moduli* and implement some viable Inflationary Scenario

▲ At **String Theory** level: ▲

▲ Large Number of CY of **Compactifications**

Numerous choices of fluxes

$\Rightarrow$

Enormous Number of String Vacua ($\sim 10^{500}$)

Each one of them defines a possible Effective Theory to describe low energy physics

$\Downarrow$

String Landscape

However, most (if not all) of these vacuua predict negative cosmological constant Λ *(Anti-de-Sitter space).*

$$\Downarrow$$

▲ **Long standing Question** ▲

▲ Are there any de Sitter vacua in the **Landscape**?

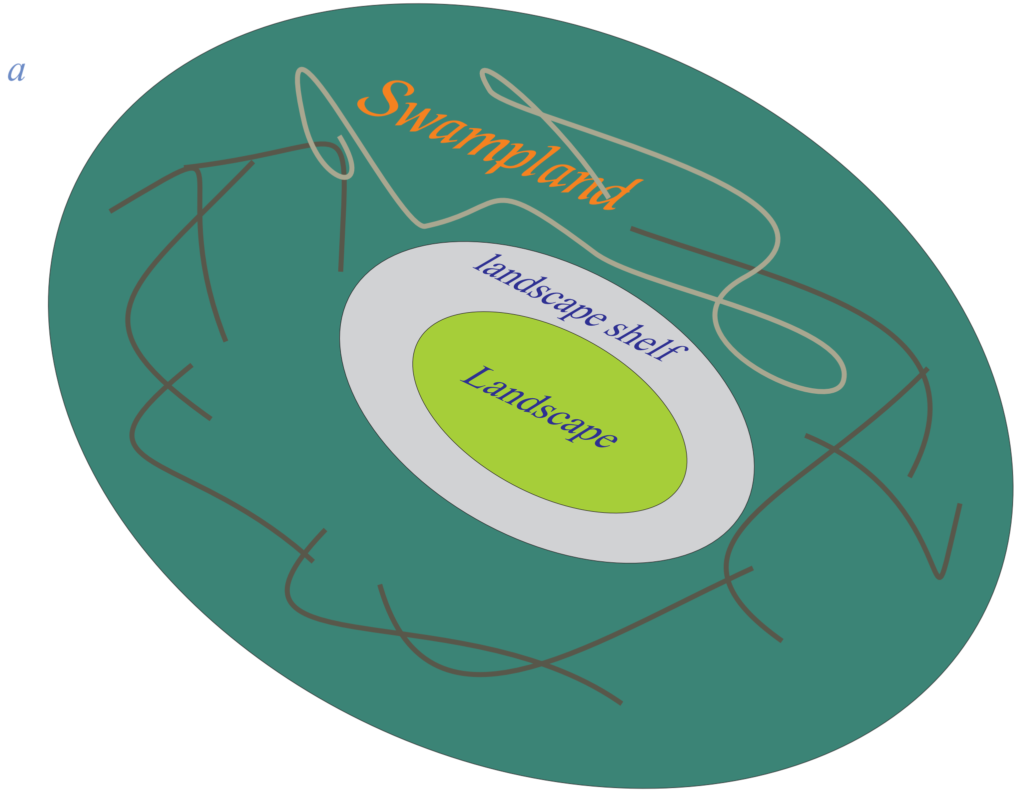
... If answer is Yes...they are...

$\Rightarrow$ *Certainly Scarce* $\Leftarrow$

Zeitgeist:

$\Rightarrow$**...according to recent claims...** $\Rightarrow$

All dS vacua fall into the $\mathcal{SWAMPLAND}$!



String landscape is surrounded by a vast swampland of inconsistent field theories of dS vacua... (according to recent conjectures...)

Grayzone populated by 'stringy' dS vacua not unanimously adopted

$\mathcal{R}$easonable $\mathcal{P}$roject

- Propose a solution to the Moduli Stabilisation problem
- Examine whether a dS vacuum exists in String Theory

- If yes:

examine cosmological implications such as inflation.

★ **Moduli** Space (*notation*)

▲ Graviton, dilaton $\cdots$

$$g_{\mu\nu},\ \phi, \cdots$$

▲ Scalar, $\cdots$

$$\mathbf{C_0},\ \cdots$$

1. ▲ C_0, ϕ $\rightarrow$ *combined to* **axion-dilaton** *modulus:*

$$S = C_0 + i\,e^{-\phi} \equiv C_0 + \frac{i}{g_s}$$

2. z_a : *Complex Structure (CS)* moduli (**shape**)

3. T_i : Kähler (**size**) $\leftrightarrow$ $J = g_{ij} dz^i \wedge dz^j$

Basic 'ingredients' of Effective String Model:

1) Superpotential $\mathcal{W}$

and

2) Kähler potential $\mathcal{K}$

▲ The Superpotential $\mathcal{W}_0$ ▲

$\mathcal{W}_0$ is of fundamental importance in EFT. It provides couplings between the fields (quarks, leptons, moduli, ...) which describe their interactions and masses observed in experiments etc

▲ The superpotential is a function of S and z_a, but not of the Kähler moduli T_i.

One can prove that z_a *and* S **acquire correct masses**

but!

▲ Kähler moduli $\notin \mathcal{W}_0 \Rightarrow$ remain unfixed! ▲

▲ The **classical** Kähler potential **is** *a function of the Kähler moduli which define the internal volume* $\mathcal{V} \sim \prod_i (T_i - \bar{T}_i)^a$*:*

$$\mathcal{K}_0 = -\sum_{i=1}^{3} \ln(-i(T_i - \bar{T}_i)) + \cdots \propto -\log(\mathcal{V}_{internal}) + \cdots \qquad (1)$$

and we can use it to compute the **scalar potential** *of EFT.*
However, because of the peculiar structure of $\mathcal{K}_0$ (dubbed no-scale) the so derived scalar potential is automatically zero:

$$V \;=\; e^{\mathcal{K}}(\sum_{I,J} \mathcal{D}_I \mathcal{W}_0 \mathcal{K}^{I\bar{J}} \mathcal{D}_{\bar{J}} \mathcal{W}_0 - 3|\mathcal{W}_0|^2) = 0$$

In other words, the Kähler moduli do not have a potential and therefore, their masses are completely **undetermined**!
(recall that $m_\phi{}^2 \propto \frac{\partial^2 V}{\partial \phi^2}$*)*

to epitomise:

Kähler moduli not fixed due to structure of classical theory

The missing ingredient is:

QUANTUM corrections

The **Question** is:

What kind of corrections ensure a dS vacuum?

Let a modulus τ *with classical potential* $V_{clas.}(\tau) \equiv 0$

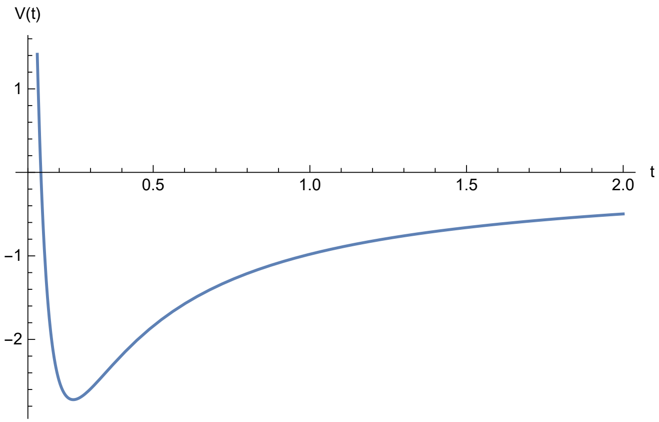
▲ Hence, any τ-dependent **perturbative** quantum corrections

$$V_{eff}(\tau) = V_{clas.} + V_{pert.}$$

must vanish for $\tau \to \infty$. If $V_{eff}(\tau \to \infty)$ vanishes from below:

$$\lim_{\tau \to \infty} V_{eff} \to 0^{-}$$

the expected shape of the potential is of the form

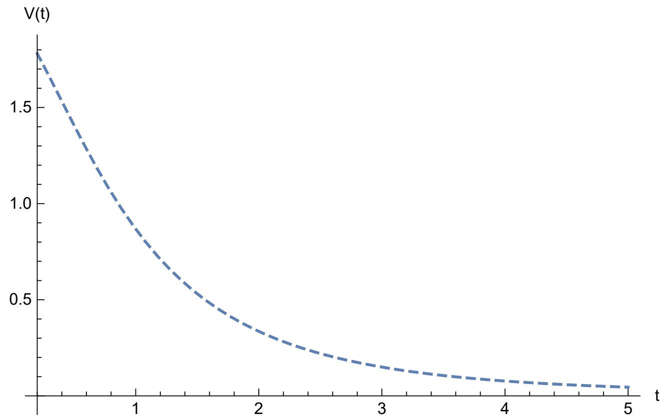


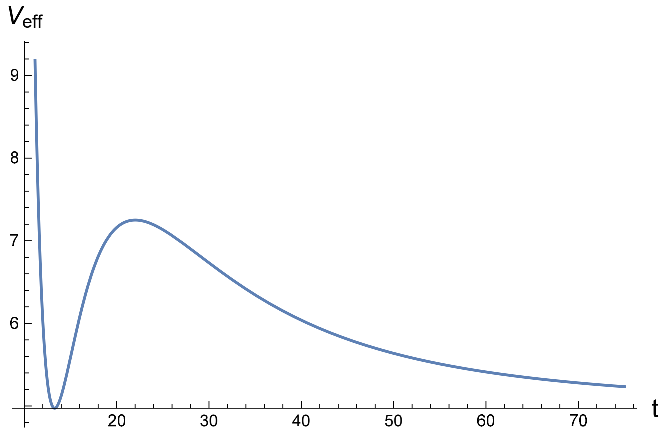
This is **Anti-**de Sitter minimum! $\Rightarrow$ **not acceptable!**

Hence, vanishing at infinity must occur from positive values

$$\lim_{\tau\to\infty} V_{eff} \to 0^{+}$$

Expected shape of the potentials:





The potential on the RHS exhibits local minimum and maximum

This suggests that V_{eff} should have **two** *competing terms*

$\mathcal{PERTURBATIVE}$

String Loop Corrections

and

The Kähler potential $\mathcal{K}$

It is known that in String Theory:

multigraviton scattering generates

higher derivative couplings in curvature R:

Leading correction in the 10-d effective action:

$$\propto \int_{M_{10}} R^4$$

Reduction on $\mathcal{M}_4 \times \mathcal{X}_6$, (with $\mathcal{M}_4$ 4-d Minkowski) induces a new Einstein Hilbert ($\mathcal{EH}$) term R Extr which lives in the six-dimensional compactified space.

It is proportional to the Euler characteristic of the manifold:

$$\chi \propto \int R \wedge R \wedge R$$

▲▲ *this $\mathcal{EH}$ term possible in 4-dimensions only*!

▲▲ Introducing 7-branes ▲▲

Localised vertices can emit gravitons and KK-excitations in 6d

$\Rightarrow$ KK-exchange between graviton vertices and $D7$-branes

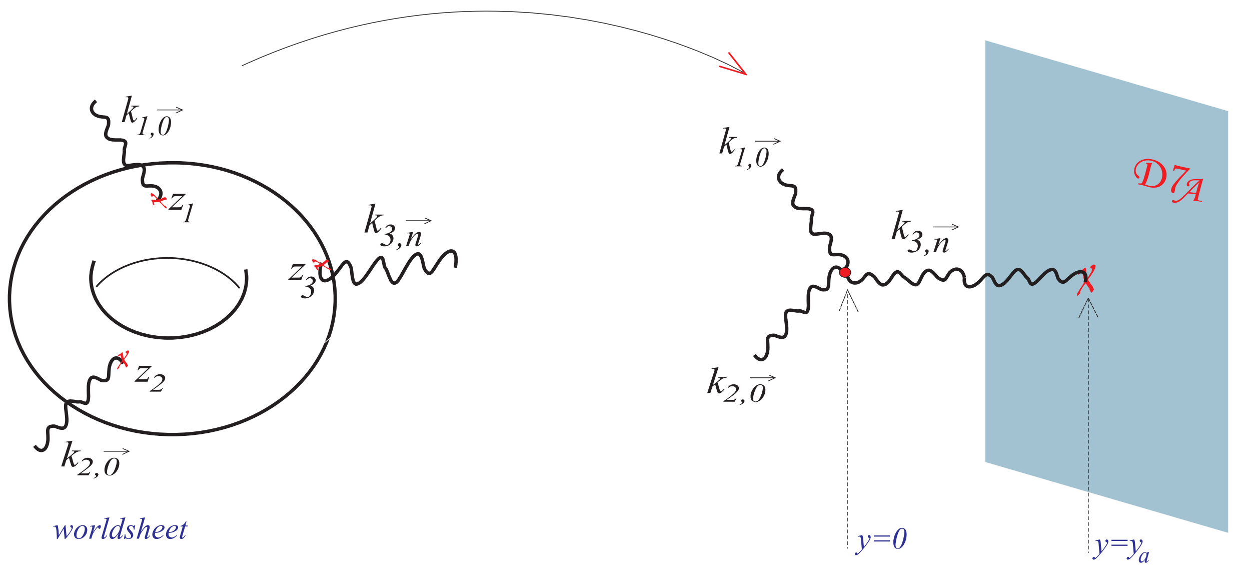


Figure: non-zero contribution from 1-loop; 3-graviton scattering.

Corrections

Final result:

$$\frac{1}{(2\pi)^3}\int_{M_4\times\mathcal{X}_6} e^{-2\phi}\mathcal{R}_{(10)} + \frac{4\zeta(2)\chi}{(2\pi)^3}\int_{M_4}(1+\sum_{i=1,2,3} e^{2\phi}T_i\log(R_\perp^i)\mathcal{R}_{(4)} \ ,$$

The geometric Set Up

Intersecting D7 membranes immersed in higher dimensions

Stabilisation requires three intersecting $D7$s! (at least)

▲ Kähler potential including loop corrections:

$$\mathcal{K} = -2\ln\{\mathcal{V} + \xi + \gamma\ln(\mu\mathcal{V})\}$$

Induced scalar potential of EFT:

$$V_{\text{eff}} \propto \gamma\frac{\ln(\mu\mathcal{V}) - 4}{\mathcal{V}^3} + \frac{d}{\mathcal{V}^2}$$

Extrema at $dV_{\text{eff}}/d\mathcal{V} = 0 \Rightarrow (w \propto \log\mathcal{V})$*:*

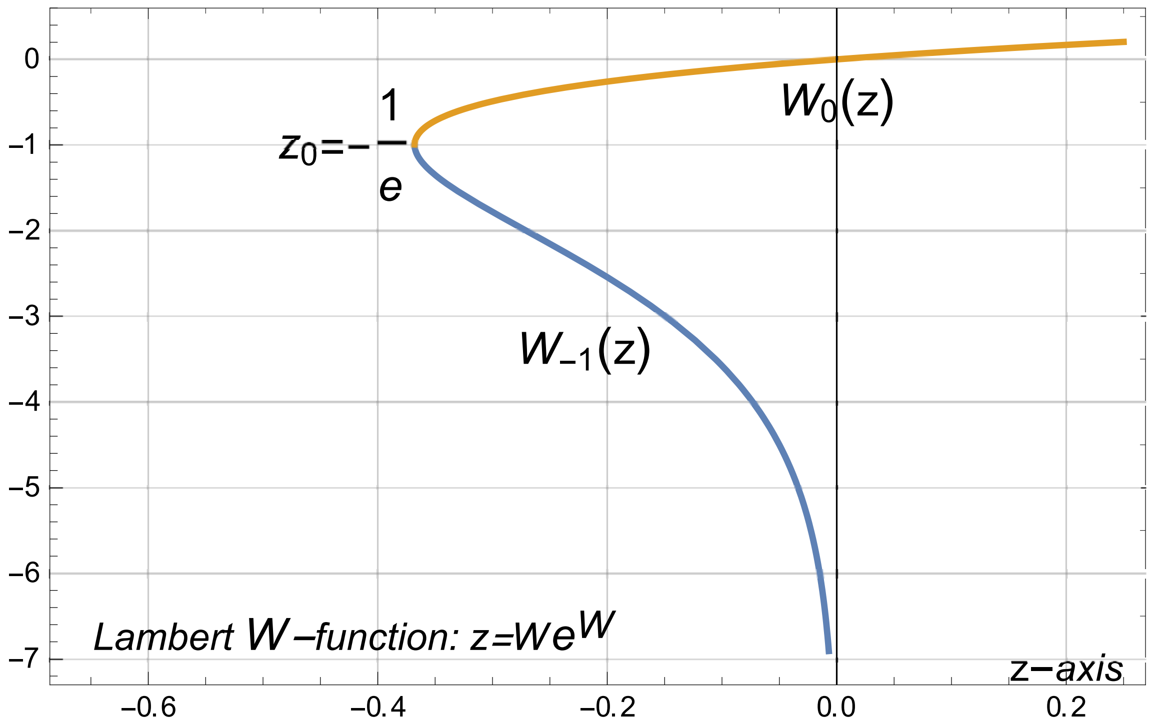
$$we^w = z$$

Solution $\rightarrow$ *double-valued Lambert* W*-function*:

$$w \Rightarrow W(z)$$

$\Rightarrow$ determines values of $\mathcal{V}$-modulus at the extrema of V_{eff}.

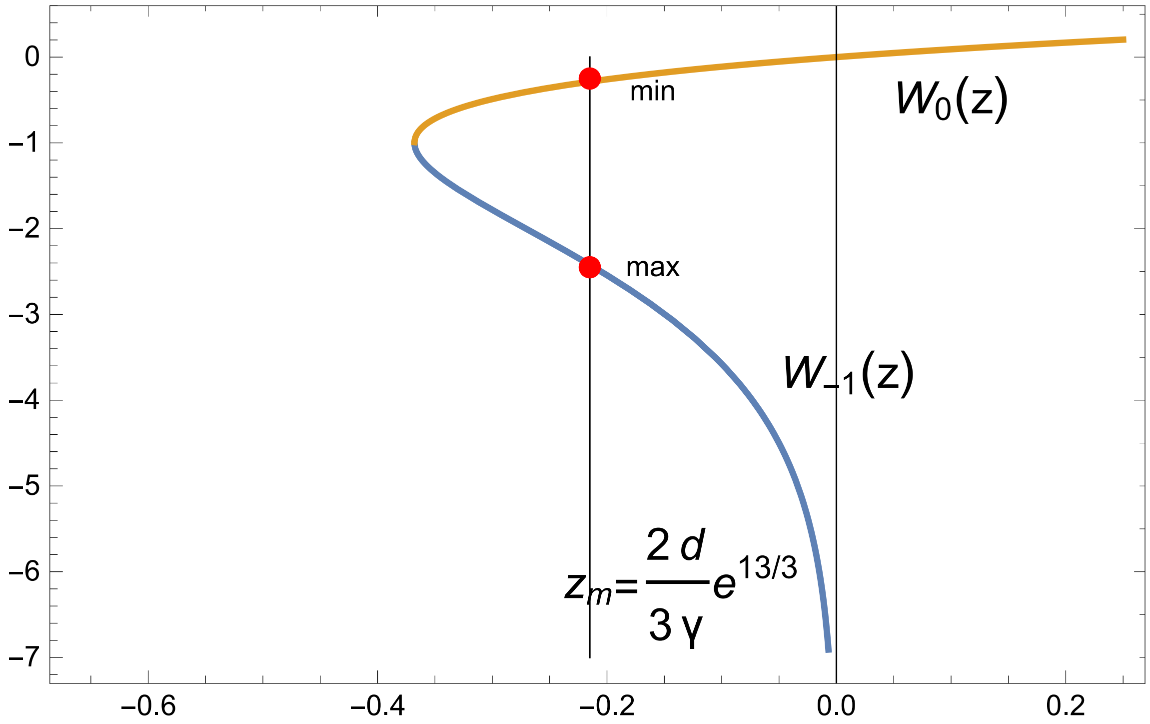
The two branches of the Lambert function $W_0(z)$ and $W_{-1}(z)$



- Real W_0, W_{-1} values $\forall z \geq z_0 = -e^{-1}$.
- Double values for $z \leq 0$.
- We need two extrema (max and min), hence

$$-e^{-1} < z < 0$$

vertical line represents any value of $z_m = \frac{2d}{3\gamma} e^{\frac{13}{3}}$ *between* $(-e^{-1}, 0)$ *where min and max can coexist.*

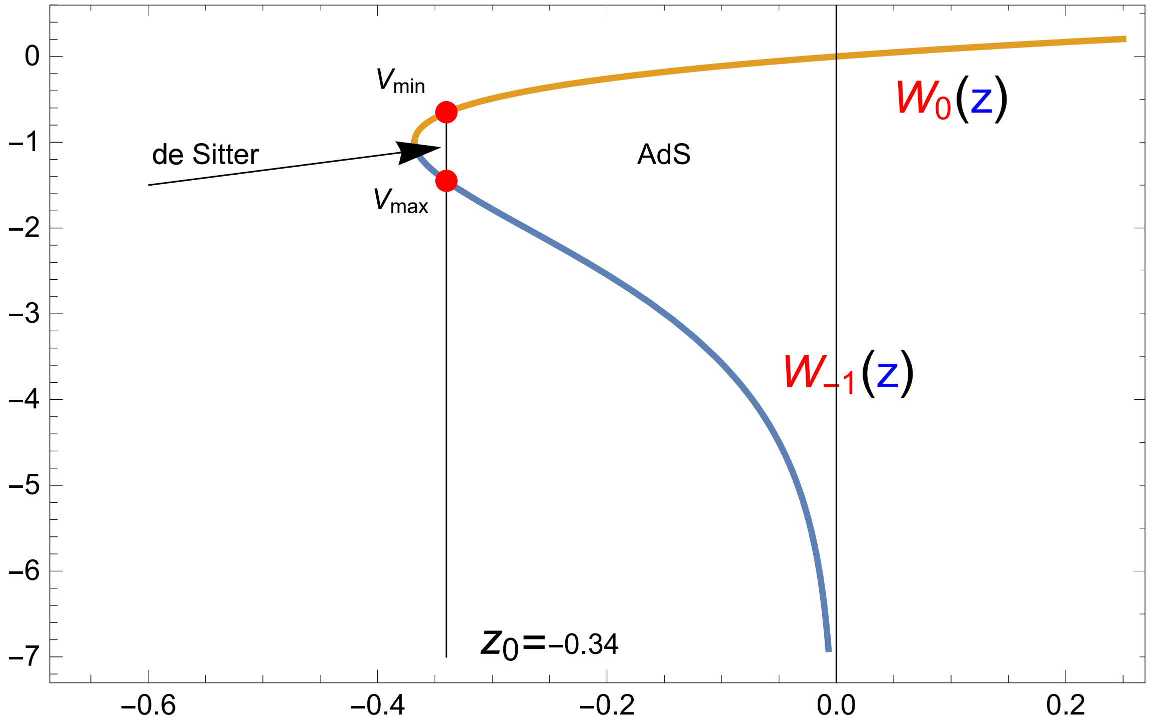


but! *requirement for de Sitter vacua puts additional restrictions*

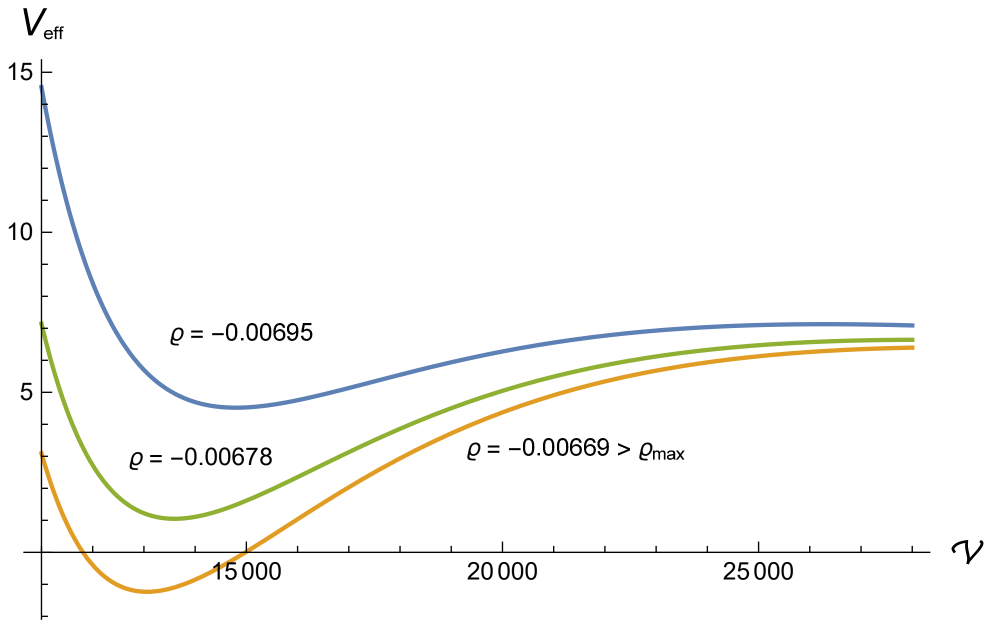
▲ de Sitter vacua ▲

minimum $V_{\rm eff} = V_F + V_{\mathcal{D}}$ *at* $\mathcal{V}_0$ *must be positive:*

$$V_{\rm eff}^{min} = \frac{\gamma}{\mathcal{V}_0^3} + \frac{d}{\mathcal{V}_0^2} > 0$$



Plot of V_{eff} vs $\mathcal{V}$ for fixed $\varrho = \frac{d}{\gamma\mu\mathcal{W}_0^2}$.
The lower curve corresponds to AdS vacuum.
At large volume, the potential vanishes asymptotically after passing from a maximum.



Non-perturbative Superpotential $\mathcal{W}$

- Adding also NP-contributions for modulus T_1

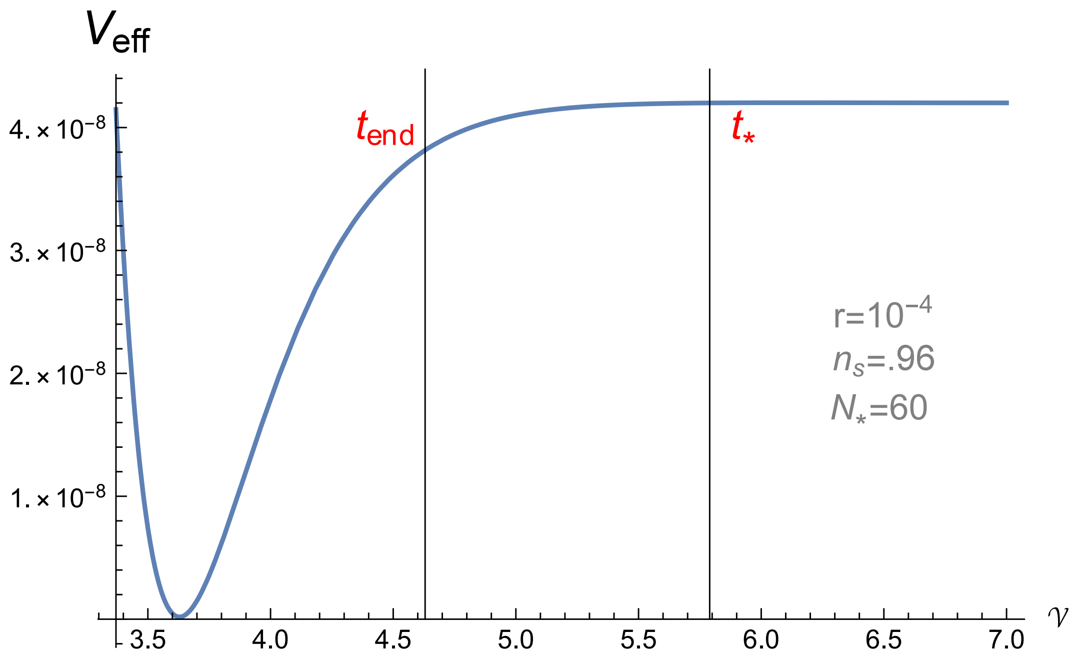
$$\mathcal{W} = \mathcal{W}_0 + Ae^{-\alpha T_1}, \quad D_{T_1}\mathcal{W} = 0$$

- Resulting scalar potential **untractable** !

For large $\mathcal{V}$ expansion:

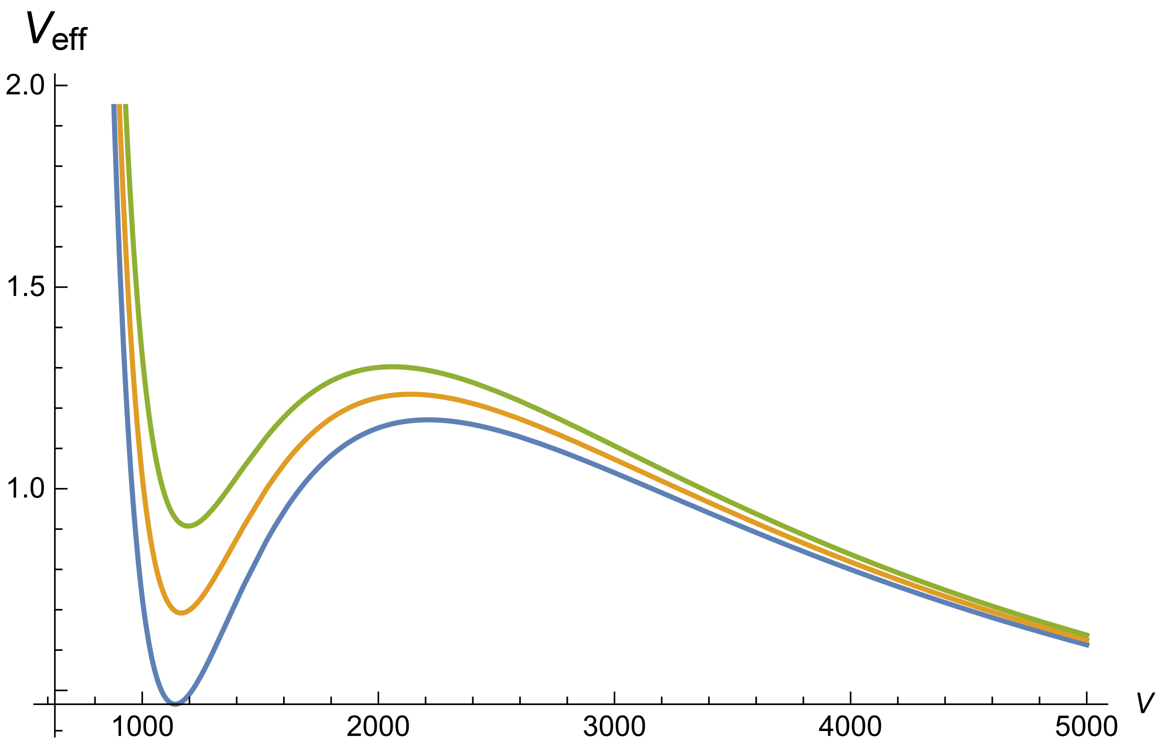
$$V_{\mathrm{eff}} \approx (\epsilon \mathcal{W}_0)^2 \left(\frac{2\xi - \mathcal{V} + 4\gamma(\log(\mathcal{V}) - 1)}{4\mathcal{V}^3} \right) + \mathrm{D-terms} \qquad (2)$$

INFLATION

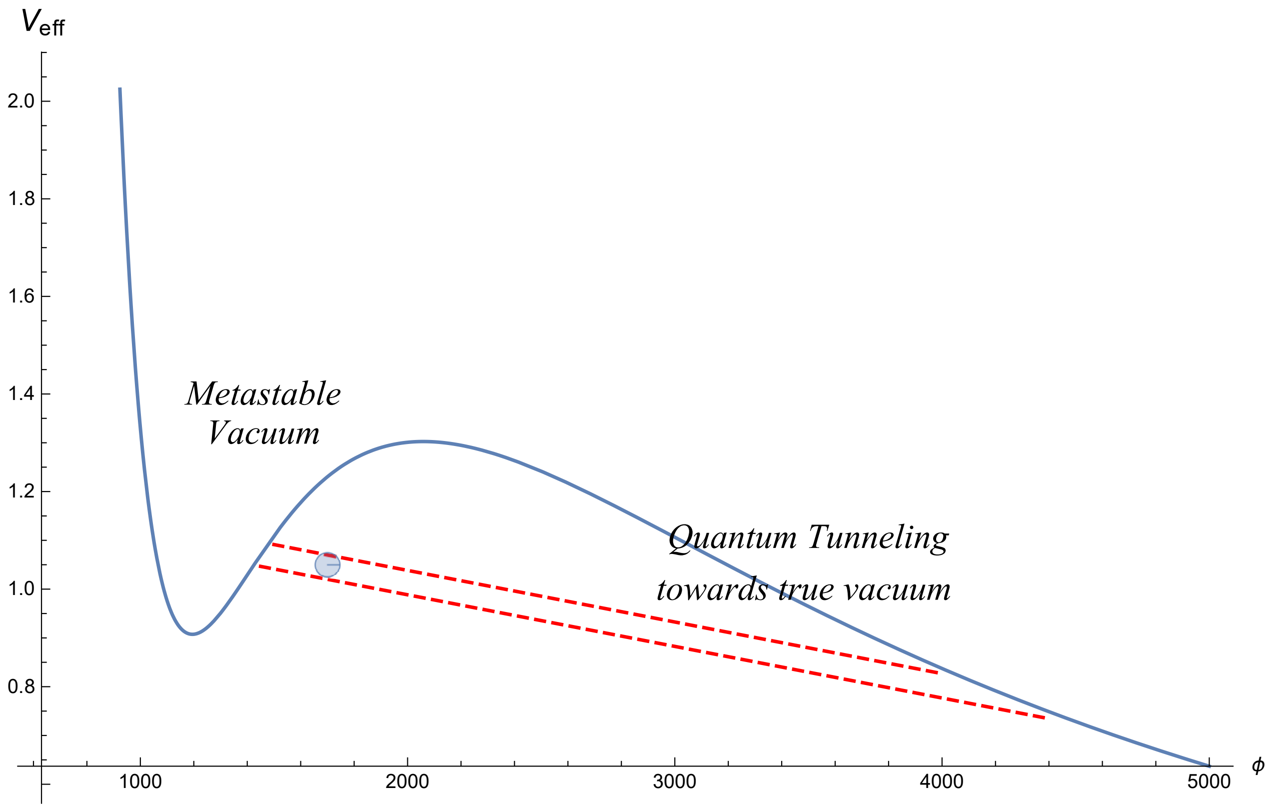


Inflation can be described by a scalar field $\phi \sim \log \mathcal{V}$ *rolling down from the top of the potential,* $\ddot{\phi} + 3H\dot{\phi} + 3V'(\phi) = 0$. *A scenario called slow-roll inflation requires a flat region. Inflation occurs between* t_* *and* t_{end} *and stops when the slope becomes steep.*

but, this is a Metastable de Sitter minimum



Quantum corrections fade out as $\mathcal{V} \to \infty$ *and* $V_{\rm eff} \to 0$.
Hence the above cannot be the true minimum.



According to the proposed scenario we live in a metastable minimum. The true vacuum is reached by the inflaton field penetrating the barrier in sufficiently long time.

Conclusions

★ *Accelerated Expansion of Universe and other Cosmological Observations impose strict requirements on String Theory and Particle Physics*

★ Data rule out most of Effective Fields Theories
(the so called String Landscape)

★ *Theory at classical level fails to fix moduli problem and create today's picture of the Universe*

★ This can happen when appropriate Quantum corrections are incorporated
(which require a geometric configuation of Strings & Branes)

⋆ Reasonable Question? ⋆

If de Sitter vacua so scarce is String Theory, why still looking for them there?

⋆ Lamppost Effect? ⋆

⋆ Thank you for your attention ⋆

based on works with:

I. Antoniadis, Y. Chen, 1803.08941, (EPJC) 1909.10525 (JHEP)

I. Antoniadis, O. Lacombe 2007.10362 (EPJC)

V. Basiouris 2007.15423 (PLB)