SPACE-TIME FINITE ELEMENT METHODS FOR A CLASS OF QUASILINEAR PARABOLIC PROBLEMS

IOANNIS TOULOPOULOS �,*

ABSTRACT

In this talk, we present continuous finite element methods for solving quasilinear parabolic problems of the type $\partial_t u - \operatorname{div} A(\nabla u) = f$, in $Q_T = \Omega \times [0, T]$, in space and time simultaneously. We focus mainly on the problems with $A(\mathbf{a}) = (\varepsilon + |\mathbf{a}|)^{p-2}\mathbf{a}$. In our approach we consider the time variable t as just another variable, say, x_{d+1} if $x_1, \ldots, x_d$ are the spatial variables, and the time derivative as a strong convection in the direction x_{d+1}. Multiplying the associated PDE by a test function depending on spatial and time variable and then applying integration by parts we lead to the space-time formulation. We treat numerically the discretization of the time derivative in a stable way by using Streamline-Upwind Petrov-Galerkin (SUPG) techniques. Using the results presented in [1] and [2], we derive discretization error estimates taking into account that the exact solution can exhibit anisotropic regularity behavior, i.e., different regularity properties with respect to the time and to the space directions. In the last part of the talk, we present numerical results which confirm the theoretical convergence rate estimates. This talk is based on [3]. This work has been supported by the project JKU-LIT-2017-4-SEE-004.

References

[1] L. Diening, M. Růžička. *Interpolation operators in Orlicz - Sobolev spaces*. Numer. Math. 107, (2007) 107 - 129.

[2] S. Nakov and I. Toulopoulos. *Error estimates for finite element methods for quasilinear elliptic problems*. Computers and Mathematics with Applications 104 (2021) 87 - 112

[3] I. Toulopoulos. *Space-time finite elements for quasilinear parabolic problems*, under review 2020.

�Institute of Computational Mathematics Johannes Kepler University Linz,, *University of Western Macedonia GREECE, Department of Informatics, Fourka Area, 52100 Kastoria, GREECE,, email: itoulopoulos@uowm.gr