

Regularity for the prescribed Lorentzian mean curvature equation with charges

Luciano Mari

Università degli Studi di Torino

luciano.mari@unito.it

Abstract

In electrostatic Born-Infeld theory, the potential u_ρ generated by a charge distribution ρ on \mathbb{R}^m (typically, a measure) is required to minimize the action

$$\int_{\mathbb{R}^m} \left(1 - \sqrt{1 - |D\psi|^2}\right) dx - \langle \rho, \psi \rangle$$

among functions with $|D\psi| \leq 1$ and a suitable decay at infinity. Formally, the Euler-Lagrange equation (\mathcal{BI}) prescribes ρ as the Lorentzian mean curvature of the graph of u_ρ in Minkowski spacetime \mathbb{L}^{m+1} , making contact with General Relativity. For instance, if ρ is a finite sum of Dirac deltas, then the graph of u_ρ is a maximal hypersurface with singularities in \mathbb{L}^{m+1} . While the existence/uniqueness of u_ρ follows from standard variational arguments, the singularity of the Lagrangian density when $|D\psi| = 1$ makes the solvability of (\mathcal{BI}) an open problem for most classes of measures ρ . In this talk, I will report on a recent joint work with J. Byeon, N. Ikoma and A. Malchiodi, where we study the solvability of (\mathcal{BI}) and the regularity of u_ρ under mild conditions on ρ . One of the main problems is the possible presence of light rays in the graph of u_ρ , which will be discussed in detail.